Endogenous Dampening in Task Based Models

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March 2026

Abstract

This note derives conditions under which an endogenous automation frontier eliminates task bottlenecks in a CES task-based model, despite complementarities across tasks. An expanding AI task share can offset the drag from unbalanced factor growth, and the key condition admits a clean form: automation scales without bottleneck so long as the slope of the AI productivity gradient is less than the elasticity of substitution between AI and labour.

1 Introduction

This note discusses the conditions that give rise to 'endogenous dampening' in task based models. While the possibility of bottlenecks is a well-know feature of these models, here I refer to *dampening* as the process that gives rise to these bottlenecks. Specifically, endogenous dampening occurs when the elasticity of the output of a production function, with respect to a growing input, is *declining*. This is in contrast to a production function that features endogenous *amplification* where the weight that the system puts on a growing factor is increasing over time. Regarding automation, Aghion et al. (2019) point out that this dampening can be overcome if automation of new tasks occurs fast enough. Davidson et al. (2026) provide a sufficient condition for dampening not to occur, namely that $-\varepsilon \geq 1 - \sigma$ where ε is (roughly) the elasticity of fraction of tasks automated with respect to the relative abundance of AI to human labor and σ is the elasticity of substitution between tasks.

To illustrate, consider the (reduced form) production function

$$Y \propto K^{\gamma(K/L)} L^{1-\gamma(K/L)} \tag{1}$$

where γ is the capital-output elasticity which we allow to be a function of the capital-labor ratio. We can recover a CES production function from this reduced form by setting elasticities according to:

$$\gamma(k) = \frac{1}{\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma}} k^{\frac{1-\sigma}{\sigma}} + 1} ;$$

where $\alpha^{\frac{1}{\sigma}}$ is the CES weight on capital. If $\sigma = 1$ then this reduces to the Cobb Douglas form:

$$\gamma(k) = \alpha .$$

Therefore, we can see that if K is growing faster than L (and α is constant) then $\gamma(k)$ is declining while $\sigma < 1$ and growing while $\sigma > 1$. Since capital is a function of output, then this environment features endogenous dampening when $\sigma < 1$ and endogenous amplification when $\sigma > 1$; the former case being that more capital makes future capital less productive and the latter case being that more capital makes future capital more productive. We can see that dampening is exactly the process by which complementarities can lead to bottlenecks, with $\gamma(k) \to 0$ as $k \to \infty$ for $\sigma < 1$, so additional capital growth has no impact on output.

Aghion et al. (2019) notes that in the context of automation we may simultaneously have an increasing capital stock and an increasing production weight on capital α. Therefore the question becomes whether $\frac{1-\alpha}{\alpha}$ decreases faster than $(\frac{K}{L})^{1-\sigma}$ increases. Davidson et al. (2026) hence label $-\varepsilon \geq 1 - \sigma$ as a sufficient condition for such a production function to avoid bottlenecks, where ε how much $\frac{1-\alpha}{\alpha}$ changes with a percent change in the capital to labor ratio. Aghion et al. (2019) also note that the application of task based models (eg. Acemoglu and Restrepo (2018)) can endogenize the relationship between α and the capital labor ratio pinning down a specific ε; with α representing the fraction of tasks that are automated which will increase with the capital-labor ratio as capital becomes relatively cheap making it more desirable to employ in production. However, they chose to avoid this exercise due to uncertainties related to the task productivity assumptions on capital that would ultimately pin down the elasticity of automation to changes in the capital-labor ratio. Given a recent interest in attempting to model the economic impacts of AI, and some attempts to do this with an endogenous automation frontier,[1] I pick up this exercise suggested by Aghion et al. (2019). That is, if we assume some profile of AI productivity on economic tasks does that profile suggest that an abundance of AI labor is dampening or amplifying?

[1] For example, see Kwa (2026).

2 The Model

2.1 Task-Based Production

Following Davidson et al. (2026), suppose we are interested in the aggregate effective labor, $\hat{L}$ that combines AI labor (L_{AI}) and human labor (L_H) according to the Cobb-Douglas aggregator:

$$\hat{L} \propto L_{AI}^{f} L_{H}^{1-f} \tag{2}$$

where $\hat{L}$ is the size of the effective labor force. Thus the focus for Davidson et al. (2026) is deriving a condition on a set of f terms that lead to explosive growth.

Here the assumption of Cobb-Douglas aggregation means that the elasticity of effective labor with respect to AI labor is constant for a fixed level of automation:

$$\frac{\partial \log \hat{L}}{\partial \log L_{AI}} = f \ . \tag{3}$$

It is ultimately the size of this elasticity that will be important for generating explosive growth. Therefore, the assumption that these automation levels are constant becomes particularly useful in the identification of explosive growth thresholds.

Of course, there are many reasons to expect that a Cobb Douglass aggregation of human and AI labor should not be the correct way to think about automation. Task complementarities mean that slow growing human labor can bottleneck availability of effective labor in the face of abundant AI labor. Therefore, the standard tasks based model represents these bottlenecks using a CES aggregator over human and AI labor

$$\hat{L} \propto \left(\int_0^1 (\phi(j) L_{AI,j} dj)^{\frac{\sigma-1}{\sigma}} dj) + \int_0^1 (b L_{H,j} dj)^{\frac{\sigma-1}{\sigma}} d_j\right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where $\phi'(j) < 0$. Cost minimization implies that there exists some f such that for all $j \leq f$ the firm produces using AI labor, and that

$$\frac{w}{\phi(f)} = \frac{r}{b}$$

where w is the human wage, and r is the rental rate of an AI worker. From the firms optimization problem we will have

$$L_{AI,j} = \hat{\Phi}(f,j)L_{AI} \qquad \text{where} \quad \hat{\Phi}(f,j) = \frac{\phi(j)^{\sigma-1}}{\int_0^f \phi(j)^{\sigma-1}dj} \tag{5}$$

so the AI labor allocation to task j is $\hat{\Phi}(f,j) \in (0,1)$ (and $\int_0^1 \Phi(f,j)dj = 1$) and human labor is spread evenly across tasks $j \in (f,1]$ Thus we can rearrange the production function

$$\hat{L}_i \propto (f^{\frac{1}{\sigma}} L_{AI}^{\frac{\sigma-1}{\sigma}} \int_0^f (\phi(j)\hat{\Phi}(f,j))^{\frac{\sigma-1}{\sigma}} dj) + (1-f)^{\frac{1}{\sigma}} (bL_{H,j})^{\frac{\sigma-1}{\sigma}} dj)^{\frac{\sigma}{\sigma-1}} \tag{6}$$

$$= (f^{\frac{1}{\sigma}} (\Phi(f)L_{AI})^{\frac{\sigma-1}{\sigma}} + (1-f)^{\frac{1}{\sigma}} (bL_{H,j})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{7}$$

for, the CES mean of task productivities, $\Phi(f) = \left(\frac{1}{f}\int_0^f [\phi(j)^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}}$. Now, supposing f is fixed, we have the response of effective labor to an increase in AI labor:

$$\varepsilon \equiv \frac{\partial \log \hat{L}}{\partial \log L_{AI}} = \frac{1}{x^{\frac{1}{\sigma}} \left(\frac{\Phi(f)L_{AI}}{bL_H}\right)^{\frac{1-\sigma}{\sigma}} + 1} \tag{8}$$

where $x \equiv (1-f)/f$. Now, note that $\Phi(f)$ captures the average productivity of AI labor across the tasks weighted by the amount of AI labor allocated to each task. Hence the change in Φ compared to the change in the marginal productivity, ϕ, is relatively slow. Therefore, I just assume Φ is constant. Though changes in the AI labor productivity across tasks will still be important, just not in their effects on the average productivity, rather in their effects on the marginal AI productivity.

2.2 Endogenous Automation

Above we assumed that the level of automation was set by a cost minimization choice. Namely, for an interior amount of automation the share of task that are automated, f, is pinned down by

$$\frac{w}{\phi(f)} = \frac{r}{b}$$

Here I pin down w and r to get a closed form for f. To do this, I assume that the model permits a representative cost-minimising firm that solves

$$\min_{L_{AI},L_H} rL_{AI} + wL_H \qquad \text{such that} \qquad \hat{L} \geq L \tag{9}$$

for some fixed $L > 0$. This implies human wages and AI rental rates are equal to their marginal products. Thus we have the equilibrium condition

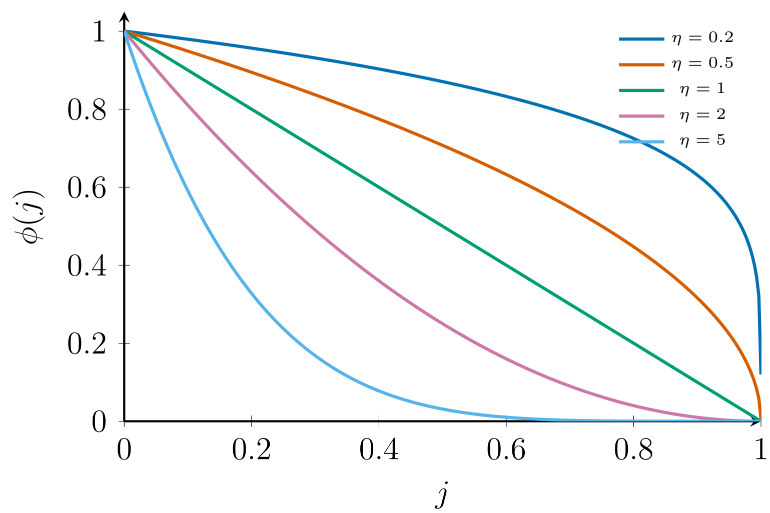
$$\begin{aligned}\phi(f) &= \frac{MP_{AI}}{MP_H} b \qquad (10)\\ &= \left(\frac{1-f}{f}\frac{L_{AI}}{L_H}\right)^{\frac{-1}{\sigma}} \left(\frac{\Phi}{b}\right)^{\frac{\sigma-1}{\sigma}} \qquad (11)\end{aligned}$$

and to get a closed form solution for f we just have to make an assumption on the form of ϕ.

Assumption 1. *$\phi(j) = a(1-j)^\eta$ for some a and $\eta > 0$.*

And I plot this function under a number of values of η in Figure 1. As we can see this give us a fair bit of flexibility in how we model the productivity of AI on different tasks in the economy.

Figure 1: AI productivity (ϕ) across tasks (j) with $a = 1$.



Therefore, under this assumption we have that

$$\begin{aligned} f &= 1 - \left(\frac{1}{a}\right)^{\frac{1}{\eta}} \left(x\frac{L_{AI}}{L_H}\right)^{\frac{-1}{\eta\sigma}} \left(\frac{\Phi}{b}\right)^{\frac{\sigma-1}{\eta\sigma}} \qquad (12)\\ \implies x^{\eta+\frac{1}{\sigma}}(1+x)^{-\eta} &= \frac{1}{a}\left(\frac{\Phi}{b}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{L_{AI}}{L_H}\right)^{\frac{-1}{\sigma}} \qquad (13)\end{aligned}$$

and from here we can take logs and derivatives to recover the elasticity of x with respect to the ratio L_{AI}/L_H.

$$\varepsilon = -\frac{1}{1+\sigma\eta f} \tag{14}$$

Combining this with the sufficient elasticity result from Davidson et al. ($-\varepsilon \geq 1-\sigma$) we can rewrite that sufficient elasticity condition as an endogenous condition on the automation frontier:

Result 1. *Under Assumption 1 the task based model with cost-minimizing automation threshold $f \in [0,1]$, there exists an automation threshold, f^*, that divides the automation frontier space across three cases:*

- *while $f < f^*$ the model is endogenously amplifying;*
- *while $f = f^*$ there is a constant weight on AI labor; and*
- *while $f > f^*$ the model is endogenously dampening;*

where

$$f^* = \frac{1}{\eta(1-\sigma)}$$

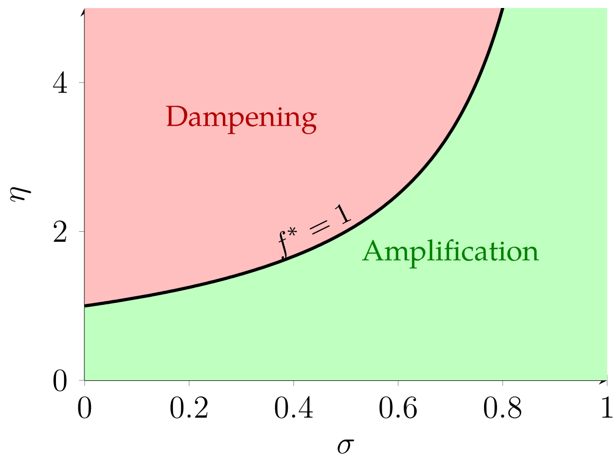
Further, we can extend this condition to identify whether the model will ever feature bottlenecks. Namely, if $f^* \geq 1$ then there is no interior f such that the model will feature interior dampening. Therefore setting $f^* = 1$ we can solve for the parameter space that sustains amplification at all times.

Result 2. *Under Assumption 1, for all $f \in [0,1]$ the task based model with cost-minimizing automation features endogenous amplification, and hence no bottlenecks, so long as*

$$\boxed{1-\sigma < \frac{1}{\eta}}$$

In Figure 1 I plot this inequality over (what I consider to be) reasonable values of η and σ. As we can see, for more substitutable tasks (σ closer to 1) we can afford to have AI being much lest productive at higher indexed tasks (modulated by η).

Figure 2: Parameter space for endogenous dampening vs amplification.



Note: The red region represents the parameter space where *eventually* the model features endogenous dampening, and ultimately bottlenecks; the green region is the space that always features amplification and never bottlenecks.

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